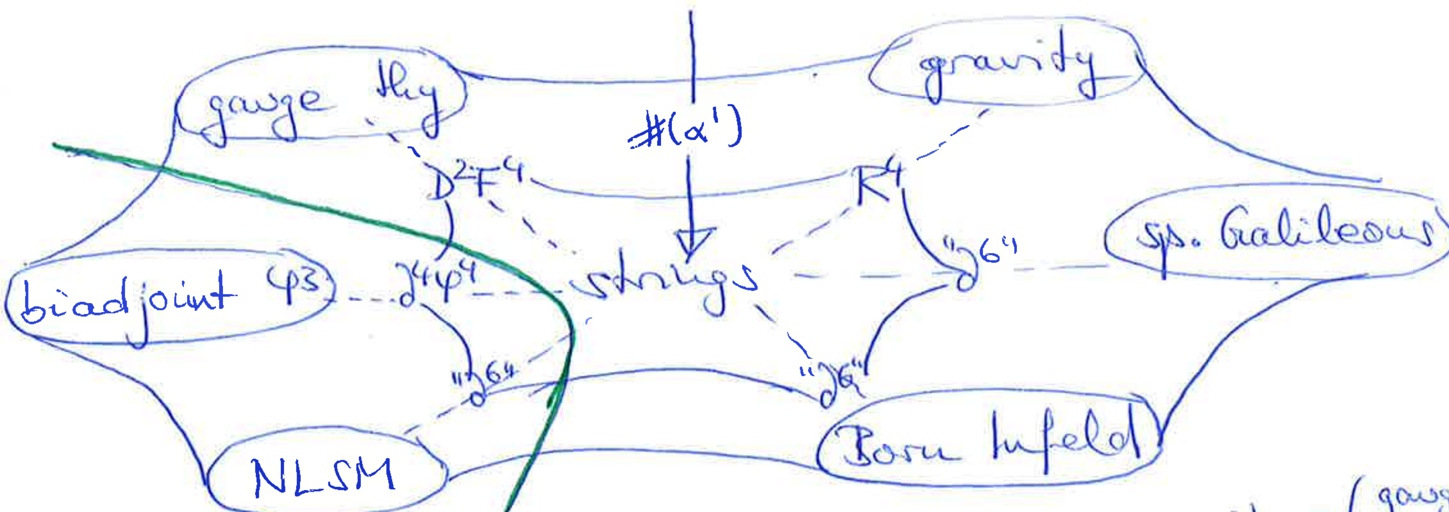
Strings tying gauge theories to gravity & scalar theories

1608.02569
1609.07078
1612.06446

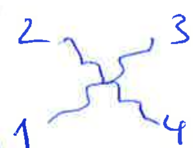
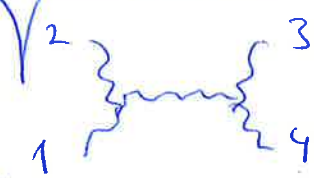
A variety of S-matrices were recently traced back to a few elementary building blocks



"Z-theory"

in the same way as gravity $= \left(\begin{matrix}\text{gauge}\\ \text{thy}\end{matrix}\right)^2$,
open strings = (gauge theory) × (Z-theory)

I) Cubic-graph formulation

$\mathcal{L}_{gauge} \Rightarrow$  + ( = $1 = \frac{(k_1+k_2)^2}{(k_1+k_2)^2}$ ) + ferm.

$f^{123}\times\left[(\epsilon_1\cdot\epsilon_2)(k_1\cdot\epsilon_3)\pm \ldots\right]$ $\sum_a f^{12a}f^{a34}(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4)$ ± perm's

$$\Rightarrow M^{g\text{-loop}}_{gauge} = \prod_{j=1}^{g}\int d^D\ell_j \sum_{\text{cubic graph } i} \frac{c_i\; n_i(\ell)}{\prod_{\text{edges } \alpha_i} k^2_{\alpha_i}(\ell)}$$

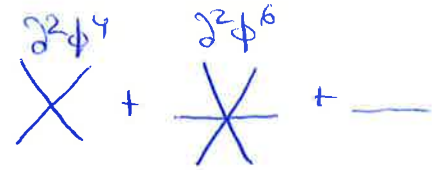
kinematics: momenta k, ℓ & (super-)polarizations ϵ from $\mathcal{L}_{YM}$ & 

color: dress a —— b w. δ^{ab} and $b, a \succ c$ with f^{ijk}_{abc}

Cast of characters:

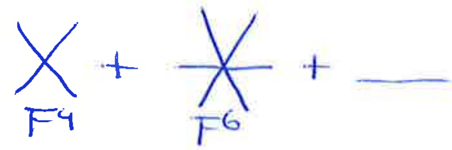
$$\mathcal{L}_{\varphi^3} = \frac{1}{2}\partial_\mu \varphi^{a|\tilde{b}}\partial^\mu\varphi^{a|\tilde{b}} + \frac{g}{3} f_{abc}\tilde{f}_{\tilde{a}\tilde{b}\tilde{c}}\,\varphi^{a|\tilde{a}}\varphi^{b|\tilde{b}}\varphi^{c|\tilde{c}}$$

$$\mathcal{L}_{NLSM} = \frac{1}{2}\mathrm{Tr}\left\{\partial_\mu\phi\,\frac{1}{1-\lambda^2\phi}\,\partial^\mu\phi\,\frac{1}{1-\lambda^2\phi^2}\right\} = $$



see [Cheung, Shen 1612.00868] for cubic action

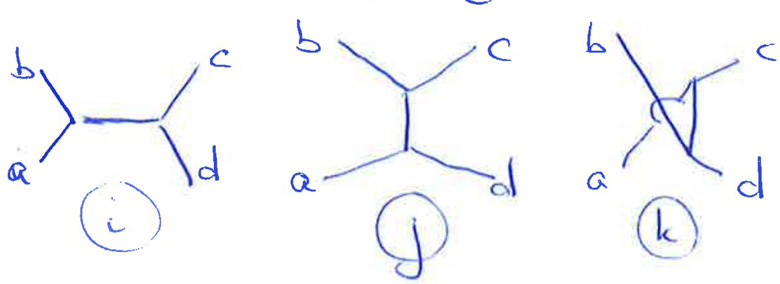
$$\mathcal{L}_{BI} = \frac{1}{\alpha'^2}\left(\sqrt{\det(\eta_{\mu\nu}-\pi\alpha' F_{\mu\nu})} - 1\right) = $$



$$\mathcal{L}_{sp.Gal} = \frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi + \sum_{m=3}^{\infty} g_m\,\chi\det\{\partial^{\mu_i}\partial_{\nu_j}\chi\}_{i,j=1,2\ldots m-1}$$

special values of couplings $g_3, g_4, g_5, \ldots$

Jacobi identity $\sum_e f^{e[ab}f^{c]de} = 0 \Rightarrow c_i - c_j + c_k = 0$ for



$\Rightarrow$ ambiguity in n_i

from

BCJ 2008/2010: $\exists\,\{n_i(\ell)\}$ with same symmetries as c_i,

$n_i(\ell) - n_j(\ell) + n_k(\ell) = 0 \quad \forall$ above $\{i,j,k\}$ & $\forall\,\ell$

Given such "BCJ representation" $\{n_i(\ell)\}$

"hard to find", usually fail

$$\Rightarrow M^{g\text{-loop}}_{\text{gravity}} = \prod_{j=1}^{g}\int d^D\ell_j \sum_{\substack{\text{cubic}\\ \text{graph}}} \frac{n_i(\ell)\,\tilde{n}_i(\ell)}{\prod_{\text{edge }\alpha_i} \ell^2_{\alpha_i}(\ell)}$$

possibly non-BCJ & different gauge thy

Chen / Du 2013 & CHY 2014 : $\exists$ 3rd cubic-graph decoration

$z_i(\ell) \longleftrightarrow$ "scalar kinematics", only k, ℓ

which can be made $z_i(\ell) - z_j(\ell) + z_k(\ell) = 0$ with Jacobi rel's

given $\{c_i, n_i(\ell), z_i(\ell)\} \Rightarrow 3\times3$ array of S-matrices

$$M^{g\text{-loop}}_{A\otimes B} = \prod_{j=1}^{g} \int d^D \ell_j \sum_{\text{cubic graph } i} \frac{(a_i \in \{c_i, n_i, z_i\})(b_i \in \{\tilde{c}_i, \tilde{n}_i, \tilde{z}_i\})}{\prod_{\text{edges } \alpha_i} k^2_{\alpha_i}(\ell)}$$

$b_i \backslash a_i$	c_i	n_i	z_i
$\tilde{c}_i$	biadjoint φ^3	gauge theory	NLSM
$\tilde{n}_i$		gravity	Born Infeld
$\tilde{z}_i$			sp-Galileons

II) Manifestly gauge-/diffeo invariant formulation

this talk: tree level only [$\exists$ progress @ loops]

motivation: no need to be creative & find BCJ n_i / z_i, no dependence on their gauge dependent form if found

- (2x) partial amplitudes (cyclic)

$$A_{\substack{\text{gauge}\\ \text{NLSM}}}(1, 2, \ldots, n) = M_{\substack{\text{gauge}\\ \text{NLSM}}} \Big|_{\mathrm{Tr}(t^1 t^2 \ldots t^n)}$$

$$m[\alpha(1, 2, \ldots, n) | \beta(1, 2 \ldots n)] = M_{\varphi^3} \Big|_{\substack{\mathrm{Tr}(t^{\alpha(1)} \ldots t^{\alpha(n)}) \\ \mathrm{Tr}(\tilde{t}^{\beta(1)} \ldots \tilde{t}^{\beta(n)})}}$$

• BCJ 2008 : from $n_i - n_j + n_k = 0$ (mere existence) $\quad z_i - z_j + z_k = 0$

$$\Rightarrow 0 = \sum_{j=2}^{n-1} k_1 \cdot k_{23\ldots j}\, A_{\text{gauge}}(2,3,\ldots,j,1,j+1,\ldots,n)$$

NLSM $\qquad k_2 + k_3 + \ldots + k_j$

$$\Rightarrow \exists\,(n-3)!\ \text{basis}\ \left[\text{e.g. } \{A_{\text{gauge}}(1,\rho,n-1,n),\ \rho\in S_{n-3}\}\right]$$

• KLT 1986 : from $n_i c_i \to n_i \tilde{n}_i$ $\qquad \rho(2,3,\ldots,n-2)$

$$\Rightarrow M_{\text{grav}} = \sum_{\rho,\tau\in S_{n-3}} A_{\text{gauge}}(1,\rho,n,n-1)\, S[\rho|\tau]_1\, \tilde{A}_{\text{gauge}}(1,\tau,n-1,n)$$

$$\equiv A_{\text{gauge}}(\cdot) \otimes_{\text{KLT}} \tilde{A}_{\text{gauge}}(\cdot)$$

perm. invariant by BCJ-rel's

order $(k\cdot k)^{n-3}$ with $S_0[2|2]_1 = k_1\cdot k_2$ (4pt) and

$$S_0[a_1,\ldots,a_x,j\,|\,b_1 b_2\ldots b_y,j,c_1,\ldots,c_z]_1 = k_j\cdot k_{1b_1b_2\ldots b_y} \times S_0[a_1\ldots a_x\,|\,b_1\ldots b_y,c_1,\ldots,c_z]_1$$

• CHY 2013 : from $n_i c_i \to c_i \tilde{c}_i$ [way to solve BCJ rel's]

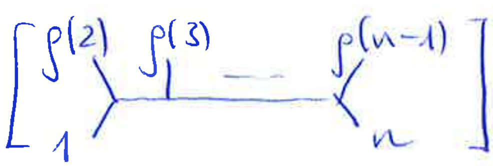
$$\Rightarrow A_{\text{gauge}}(\rho(1,2,\ldots,n)) = m[\rho(1,2\ldots n)|\cdot] \otimes_{\text{KLT}} A_{\text{gauge}}(\cdot)$$

$$\Rightarrow m[1,\rho,n-1,n\,|\,1,\tau,n,n-1] = S_0^{-1}[\rho|\tau]_1$$

• CHY 2014 : from $n_i \tilde{n}_i \to n_i z_i$

$$\Rightarrow M_{BI} = A_{\text{gauge}}(\cdot) \otimes_{\text{KLT}} A_{\text{NLSM}}(\cdot)$$

• given BCJ numerators with $(n-2)!$ Jacobi-independent

$n_{1|\rho(23\ldots n-1)|n} \equiv n\,[$  $]$

$$M_{\text{grav}} = \sum_{\rho\in S_{n-2}} A_{\text{gauge}}(1,\rho(2\ldots n-1),n)\, n_{1|\rho(2\ldots n-1)|n}$$

$$A(\tau(1\ldots n)) = \sum m[\tau(12\ldots n)\,|\,1,\rho,n]\, n_{1|\rho|n}$$

• preview: Jacobi independent z_i for NLSM

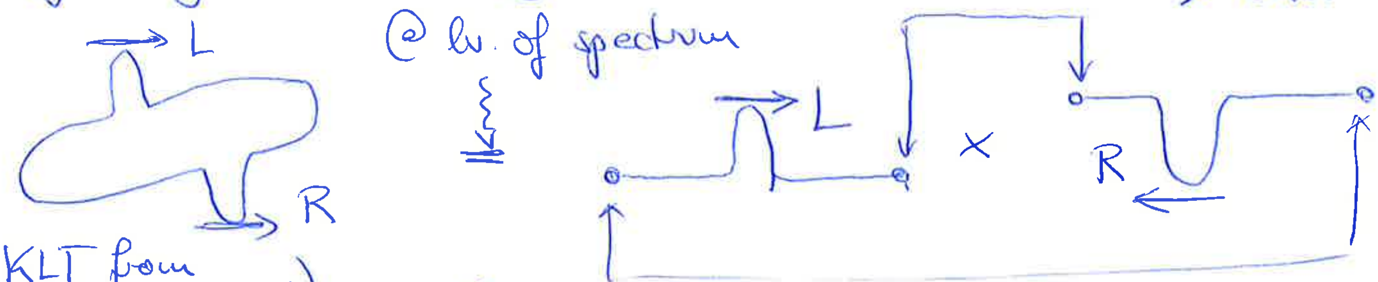
$$z_{1|\rho(23\ldots n-1)|n} = i^{n-2} S_0[\rho(23\ldots n-1)|\rho(23\ldots n-1)]_1 \sim (k\cdot k)^{n-2}$$

[moral: $S_0 \Rightarrow$ all ϕ^3 & NLSM amplitudes]

III) String-thy perspective

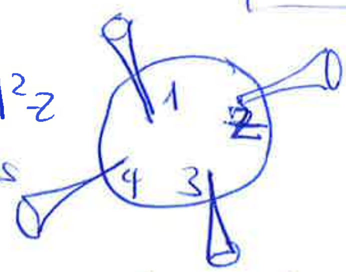
• (gravity closed strings) = $\left(\text{gauge thy open strings}/\text{color}\right)^{\otimes 2}$

@ lv. of spectrum

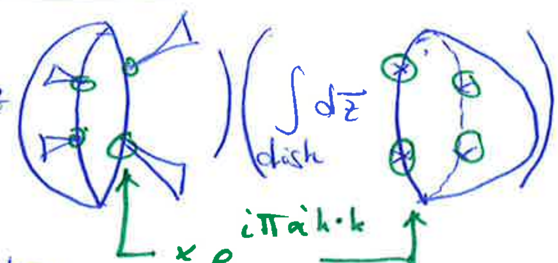


• KLT from

$$M^{tree}_{closed}(1,2,3,4) = \int_{spheres} d^2 z$$



$$= \left(\int_{disk} dz\right)\left(\int_{disk} d\bar z\right)$$



$\times\, e^{i\pi\alpha' k\cdot k}$

$$= A^{tree}_{open}(1,2,3,4)\,\sin(\pi\alpha' k_1\cdot k_2)\,A^{tree}_{open}(1,2,4,3)$$

@ $\alpha'\to 0$ { open → gauge; $\sin(\ldots)$ → $S_0[2|2]_1 = k_1\cdot k_2$; open → gauge

• BCJ from monodromy relations [Stephan's conference talk]

$$0 = \sum_{j=2}^{n-1} \sin(\pi\alpha' k_1\cdot k_{23\ldots j})\, A_{open}(2,3,\ldots j,1,j+1\ldots n)$$

• open strings as double copy [1106.2645, 1304.7267]

$$A_{open}(\rho(1,2\ldots n)) = Z(\rho(1,2\ldots n)|\cdot) \otimes_{KLT} A_{gauge}(\cdot)$$

disk integrals with 2 cycle structures $\quad z_{ij} = z_i - z_j$

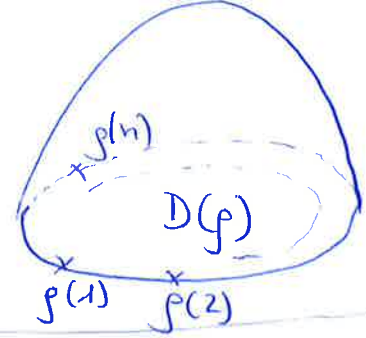
$$Z(\rho(1,2,\ldots n)|\tau(1,2\ldots n)) = \alpha'^{n-3}\int_{D(\rho)} \frac{dz_1\, dz_2 \ldots dz_n}{\mathrm{vol}\, SL_2(\mathbb{R})}\, \frac{\prod_{i<j} |z_{ij}|^{\alpha' k_i\cdot k_j}}{\tau(z_{12} z_{23}\ldots z_{n1})}$$

domain: monodromy rel's

integrand: BCJ ~~rel~~ rel's

e.g. $0 = \int_{D(\rho(1234))} dz_2 \, \frac{\partial}{\partial z_2}\left[|z_2|^{\alpha' k_1\cdot k_2} |1-z_2|^{\alpha' k_2\cdot k_3} \right]$

$= \alpha' k_1\cdot k_2 \, Z(\rho|1243) - \alpha' k_2\cdot k_3 \, Z(\rho|1423)$



• $n_i + n_j + n_k = 0$ from — from simplifying $\langle V_1 V_2 \ldots V_n \rangle$ in Pure spinor formalism [1104.5224]

$A_{open}(\tau(12\ldots n)) = \sum_{\rho\in S_{n-2}} Z(\tau|1,\rho,n)\, n_{1|\rho|n}$

cf. $A_{gauge}(\cdot) = \sum_{\rho} m[\cdot|1,\rho,n]\, n_{1|\rho|n}$

• match $\alpha' \to 0$ with $A_{gauge}(\rho) = m[\rho|\cdot] \otimes_{KLT} A_{gauge}(\cdot)$

$Z(\rho|\tau) = m_{\varphi^3}[\rho|\tau] + \zeta_2\, m_{\partial^2\varphi^4}[\rho|\tau] + \zeta_3\, m_{\partial^4\varphi^4}[\rho|\tau] + \ldots$

cf. open strings $=$ YM $+ \zeta_2$ "F^4" $+ \zeta_3$ "$D^2F^4 + F^5$" $+ \ldots$

closed strings $=$ SUGRA $+ \zeta_3$ "R^4" $+ \ldots$

monodromy

BCJ rel's

$\Rightarrow$ bicolored-scalar amplitudes with e.o.m [1609.07078]

$\Box \Phi = \Phi^2 + \alpha'^2 \zeta_2 (\partial^2\Phi^3 + \Phi^4) + \alpha'^3 \zeta_3 (\partial^4\Phi^3 + \partial^2\Phi^4 + \Phi^5) + \ldots$

• abelian open strings $t^c \to 1$

$M_{\text{abelian open}} = Z_\times(\cdot) \otimes_{KLT} A_{gauge}(\cdot)$ where $Z_\times(\rho) = \sum_{\alpha\in S_n/\mathbb{Z}_n} Z(\alpha|\rho)$

$\alpha'\to 0$ ↓ Tseytlin 80's — must have

$M_{BI} \stackrel{!}{=} A_{NLSM}(\cdot) \otimes A_{gauge}(\cdot)$

BCJ @ all α'-orders

$\Rightarrow Z_\times(\rho) = (\pi\alpha')^{n-2}\left[A_{NLSM}(\rho) + \overbrace{\zeta_2 A_{\partial^4 NLSM}(\rho) + \zeta_3 \ldots \partial^6 + \ldots}^{\text{BCJ @ all } \alpha'\text{-orders}} \right]$

[1608.02569]

- expose leading α'-orders via monodromies

$$Z_\times(\rho) = \sum_{\tau \in S_{n-2}} Z(1,\tau(2,3\ldots n-1),n|\rho)\cdot \tau\left[\prod_{j=2}^{n-1} 2i\sin\left(\frac{\pi\alpha'}{2} k_j\cdot k_{12\ldots j-1}\right)\right]$$

$\alpha' \to 0$

$$A_{NLSM}(\rho) = \sum_{\tau \in S_{n-2}} m[1,\tau,n|\rho] \times \underbrace{i^{n-2}\,\tau\left[\prod_{j=2}^{n-1} k_j\cdot k_{12\ldots j-1}\right]}$$

$$= i^{n-2} S_0[\tau(2\ldots n-1)|\tau(2\ldots n-1)]_1$$

$\Rightarrow$ BCJ master numerator for NLSM [1612.06446]